Image Restoration Using Lagrangian Minimization and Bound Selection.

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I. Abstract

In this paper an adaptively regularized image restoration technique is formulated using constrained minimization of image variations.

The problem is formulated in the following way

$$min_f \|R\boldsymbol{f}\|_{k_1}^{k_1}$$

subject to

$$\|\boldsymbol{g} - H\boldsymbol{f}\|_{k_2}^{k_2} < \varepsilon$$

Where $\boldsymbol{f}$ and $\boldsymbol{g}$ are lexicographically ordered vectors representing the restored and degraded images respectively. H and R represent the convolution matrices of the blur and regularization operator. Using this formulation different norms, including l_1 and l_2, can be used in the objective function and constraint equation.

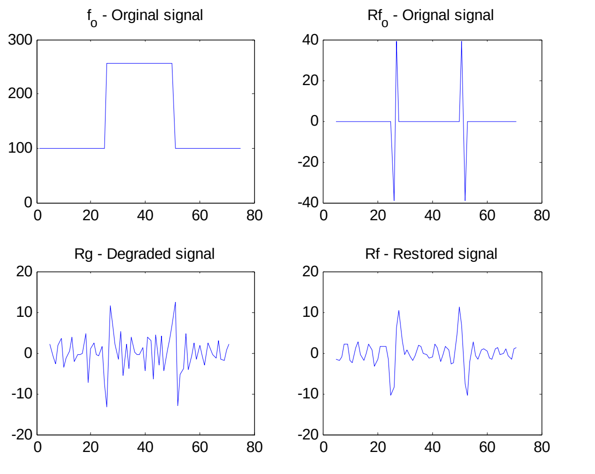

Figure 1 – Restoration of a Degraded Pulse Signal

The Lagrange multiplier method is used to develop a fast iterative restoration approach to the problem using an adaptively computed regularization parameter. These iterations are based on a Differential Multiplier Method.

A heuristic method is also proposed for finding near optimal constraint bounds. Examples of the results that were obtained using the l_2 norm for both for the objective function and the constraint equation can be found in Figures 1 and 2.

Figure 1 shows the results that were obtained from restoring a one dimensional pulse signal with a Gaussian blur of length 5 and additive zero mean Gaussian noise with a variance of 25.

Figure 2 shows the Improvement in Signal to Noise Ratio of a restored image at each iteration. These results were obtained when restoring a 256x256 cameraman image with a uniform blur of length 9 and zero mean Gaussian noise with a variance of 100. In this experiment the bound was chosen to be

$$\varepsilon = \Phi n(\sigma^2)$$

where Φ is a scaling factor, n is the number of elements in the image and σ^2 is the noise variance of the degraded image. In this example $\Phi = 0.8$ clearly gave the best results.

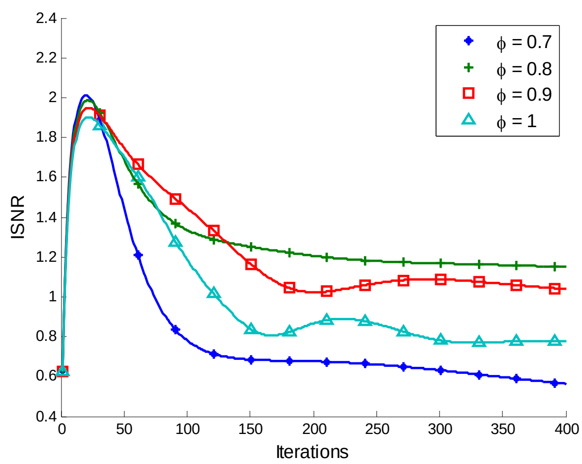

Figure 2 – Affect of Bound Selection

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