Image Restoration Using Lagrangian Minimization and Bound Selection.

M. A. Kitchener\textsuperscript{a1}, A. Bouzerdoum, IEEE Senior Member\textsuperscript{2}, S. L. Phung, IEEE Member\textsuperscript{3}

\textsuperscript{a1} School of Electrical, Computer and Telecommunications Engineering
University of Wollongong, Wollongong, NSW 2522, Australia
\textsuperscript{1}mak55@uow.edu.au, \textsuperscript{2}a.bouzerdoum@ieee.org, \textsuperscript{3}phung@uow.edu.au.

I. ABSTRACT
In this paper an adaptively regularized image restoration technique is formulated using constrained minimization of image variations.

The problem is formulated in the following way

$$\min_f ||Rf||_{k_2}^{k_1}$$

subject to

$$||g - Hf||_{k_2}^{k_1} < \varepsilon$$

Where $f$ and $g$ are lexicographically ordered vectors representing the restored and degraded images respectively. $H$ and $R$ represent the convolution matrices of the blur and regularization operator. Using this formulation different norms, including $l_1$ and $l_2$, can be used in the objective function and constraint equation.

Figure 1 shows the results that were obtained from restoring a one dimensional pulse signal with a Gaussian blur of length 5 and additive zero mean Gaussian noise with a variance of 25.

Figure 2 shows the Improvement in Signal to Noise Ratio of a restored image at each iteration. These results were obtained when restoring a 256x256 cameraman image with a uniform blur of length 9 and zero mean Gaussian noise with a variance of 100. In this experiment the bound was chosen to be $\varepsilon = \Phi n(\sigma^2)$ where $\Phi$ is a scaling factor, $n$ is the number of elements in the image and $\sigma^2$ is the noise variance of the degraded image. In this example $\Phi = 0.8$ clearly gave the best results.

The Lagrange multiplier method is used to develop a fast iterative restoration approach to the problem using an adaptively computed regularization parameter. These iterations are based on a Differential Multiplier Method.

A heuristic method is also proposed for finding near optimal constraint bounds. Examples of the results that were obtained using the $l_2$ norm for both for the objective function and the constraint equation can be found in Figures 1 and 2.

REFERENCES